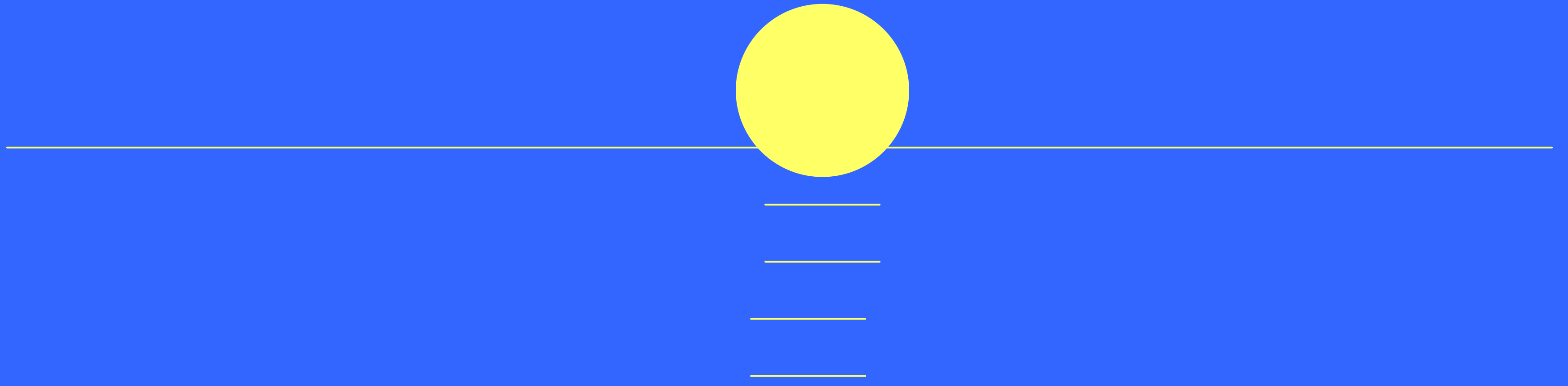
Sunset Phenomena

Roger Bailey
North American Sundial Society
Hartford Oct 1999

Walking Shadow Designs

Sunset Phenomena

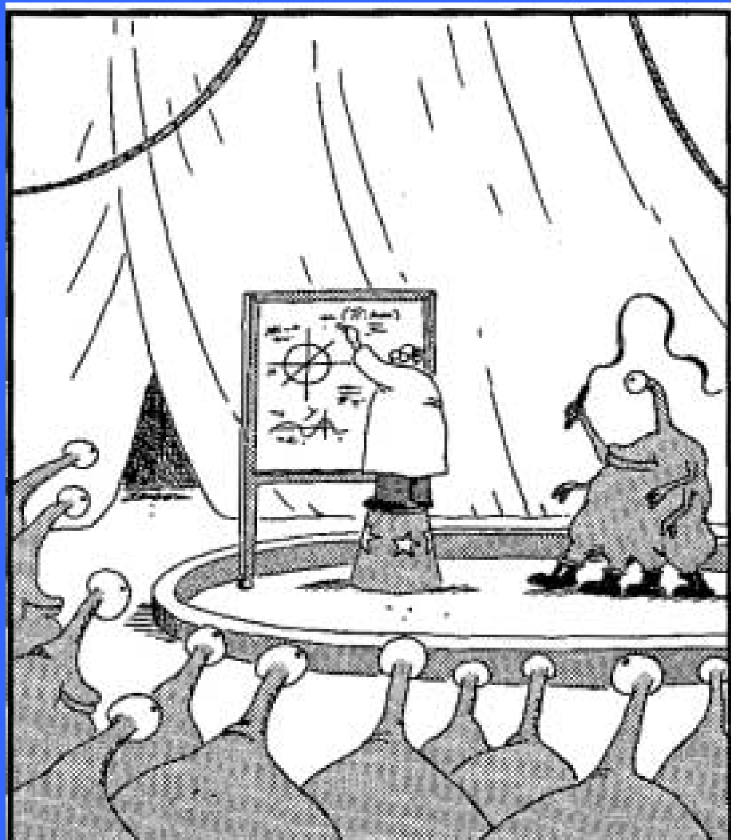


Walking Shadow Designs

Sunset Phenomena

- Simple spherical trigonometry can be used to determine for any location and day of the year
  - When sunset occurs,
  - Where the sun sets on the horizon,
  - The path of the setting sun, and
  - How long it takes for the sun to set.

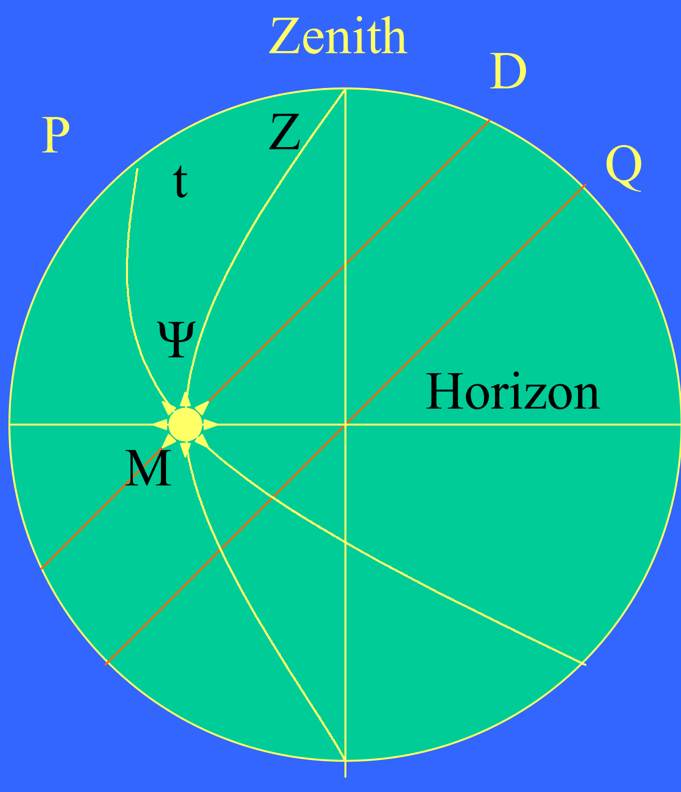
Spherical Trigonometry



Abducted by NASS aliens, Prof. Bailey is forced to write gnomic equations in center ring

- High school math but
- Non Euclidian geometry
- Parallel lines meet
- Angles in triangle > 180°
- Sides are angles
- Your position is fixed
- Sun, stars, planets move with celestial sphere

Navigational triangle



- PZ=CoL, PM=CoD, ZM=CoH
- Cosine rule: 2 sides and angle
- Cos A=Cos A Cos B+Sin A Sin B Cos c
- Sin X= Cos 90-X=Cos X
- Sin H=Sin L Sin D+Cos L Cos D Cos t
- Sin D=Sin H Sin L+Cos H Cos L Cos Z
- Sin L=Sin H Sin D+Cos H Cos D Cos Ψ
- Sine rule: Sin Z=Cos D Sin t / Cos H

Walking Shadow Designs

Navigators' Equation

- Altitude H

  $\text{Sin } H = \text{Sin } L \times \text{Sin } D + \text{Cos } L \times \text{Cos } D \times \text{Cos } t$

- Azimuth Z

  $\text{Sin } Z = \text{Cos } D \times \text{Sin } t / \text{Cos } H$

  - L = Latitude
  - D = Declination of Sun
  - t = time angle from local noon (15° / hour)

Walking Shadow Designs

Sunset Equation

- At sunrise and sunset, altitude is zero, neglecting refraction and semi-diameter
- If $H = 0$, then $\sin H = 0$

  $$\sin L \times \sin D + \cos L \times \cos D \times \cos t = 0$$

- $\cos t = -\tan L \times \tan D$
- Time of sunset is determined by the latitude and solar declination
- Sunrise is $-t$

Walking Shadow Designs

Sun as the Essence of Time

- Year: one full orbital period
  - Full Declination cycle +/- 23.5° solstices & equinoxes
  - Full Equation of Time cycle
- Day: one full rotation 360° in 24 hours
  - Time as an angle
  - 360/24 = 15°/hour = 1°/4 minutes
  - Sun at Zenith, time angle t = 0
- Clocks are wrong!

Walking Shadow Designs

Clock Time Corrections

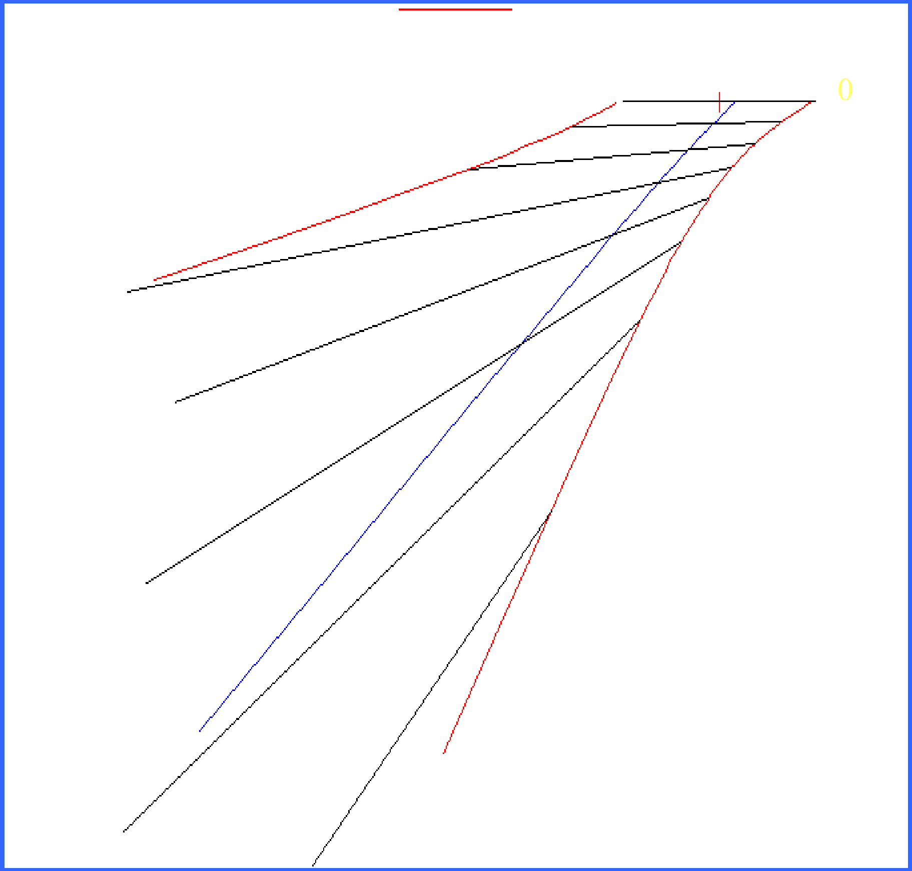
- Longitude correction
  - Mod (Longitude / 15°) as a time angle
  - Example (Canmore) 115/15 = 7 hours and 10° or 40’
- Daylight Saving Time   +1 hour in the summer
- Equation of Time Correction
- Solar noon is 1:40 pm MDST on a sunny summer afternoon in Canmore AB

Walking Shadow Designs

Sunset Time

- Sunset Equation: Cos t = -Tan L x Tan D
- Equinox: (21 March, 21 Sept) Declination = 0
  - If D = 0, Cos t = 0, t = 90° or 6 hours
  - Sunrise 6 AM, Sunset 6 PM for any latitude
- Equator: Latitude = 0
  - If L = 0, Cos t = 0, t = 90° or 6 hours
  - Sunrise 6 AM, Sunset 6 PM for any declination

Time to Sunset



- Italian Hours Sunset to Sunset
- Countdown Italian Hours gives time till sunset
- Airports: Visual Flying Rules
- RASC Alberta Star Party
  - Eccles Ranch Observatory
  - Caroline AB (52.1, 114.7)

Walking Shadow Designs

Endless Day

John Dunn
Arctic Light

Walking Shadow Designs

Polar Circles

- Sunrise Equation $\text{Cos } t = -\text{Tan } L \times \text{Tan } D$
- Polar Circle, sun does not rise
  - At noon, $t = 0$, $\text{Cos } t = 1 = -\text{Tan } L \times \text{Tan } D$
  - $\text{Tan } D = 1/\text{Tan } L = \text{Cot } L = \text{Tan } (90-L)$
  - Extreme Declination = $+/- 23\frac{1}{2}^{\circ}$
  - Extreme Latitude = $-/+ 66\frac{1}{2}^{\circ}$ $(90-23\frac{1}{2})$

Walking Shadow Designs

Sunset Location

- What is the azimuth of the setting sun?
- Cosine Rule for the Navigational Triangle
- Sin D=Sin L Sin H + Cos L Cos H Cos Z
- For H = 0, Sin H = 0 & Cos H = 1
- Cos Z = Sin D / Cos L

Walking Shadow Designs

Sunset Location

- Cos Z = Sin D / Cos L
- For the equator, L = 0, Cos L = 1

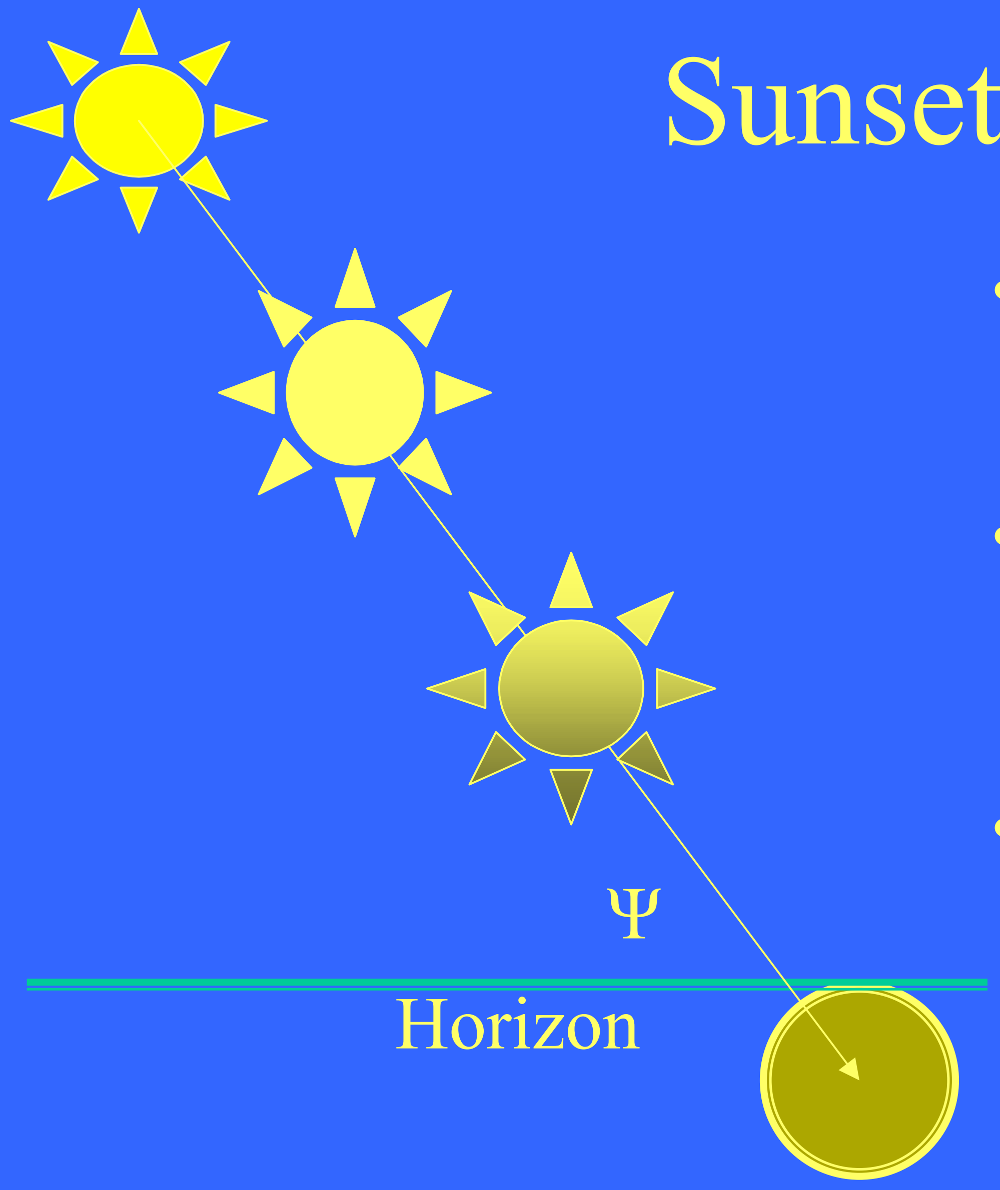
  Cos Z = Sin D = Cos (90-D) and Z = 90-D

  – The azimuth of the setting sun is the co-declination
- For equinox D = 0, Cos Z= 0, then Z = 90 for all L
  – The sun rises due east and sets due west on the equinox for all latitudes

Sunset Path

Walking Shadow Designs

Sunset Path



- Angle with horizon is Ψ

$$\cos \Psi = \sin L / \cos D$$

- At equator, L = 0

  $\cos \Psi = 0$, so

  $\Psi = 90°$ for all declinations

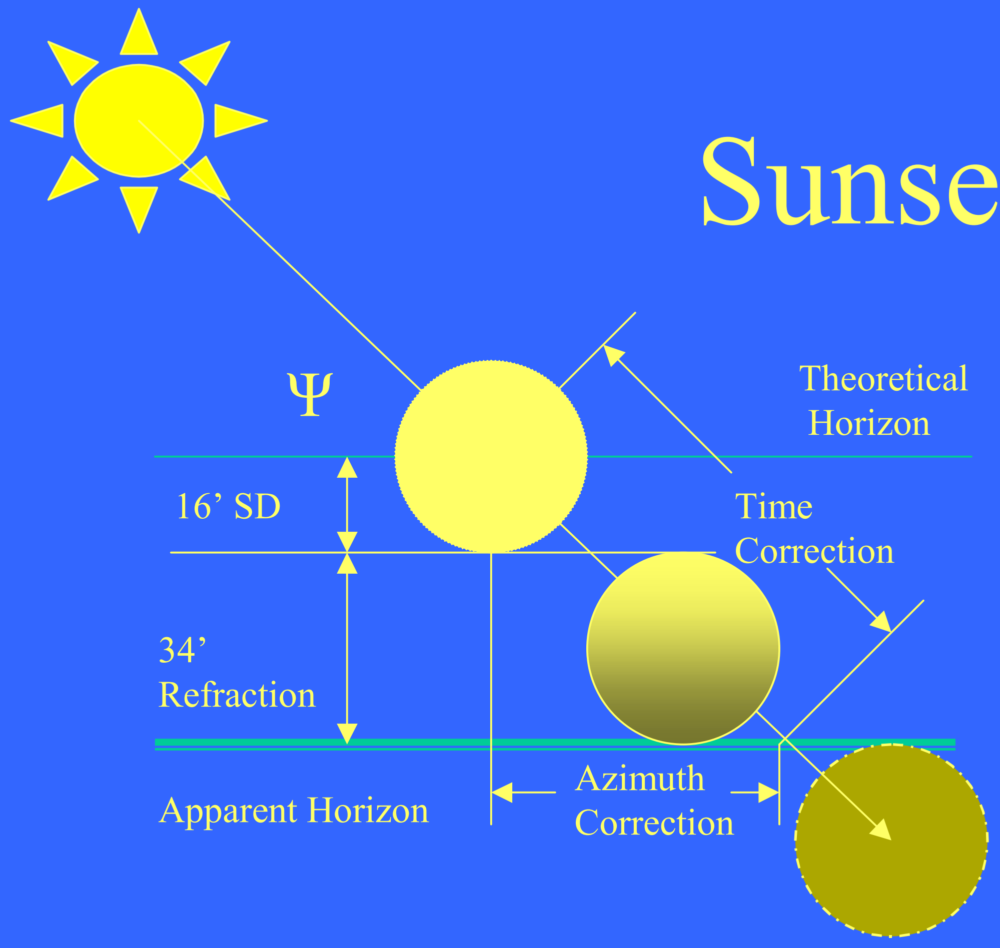
- At equinox, D =0

  $\cos \Psi = \sin L = \cos (90-L)$

  Ψ = Co-latitude

Walking Shadow Designs

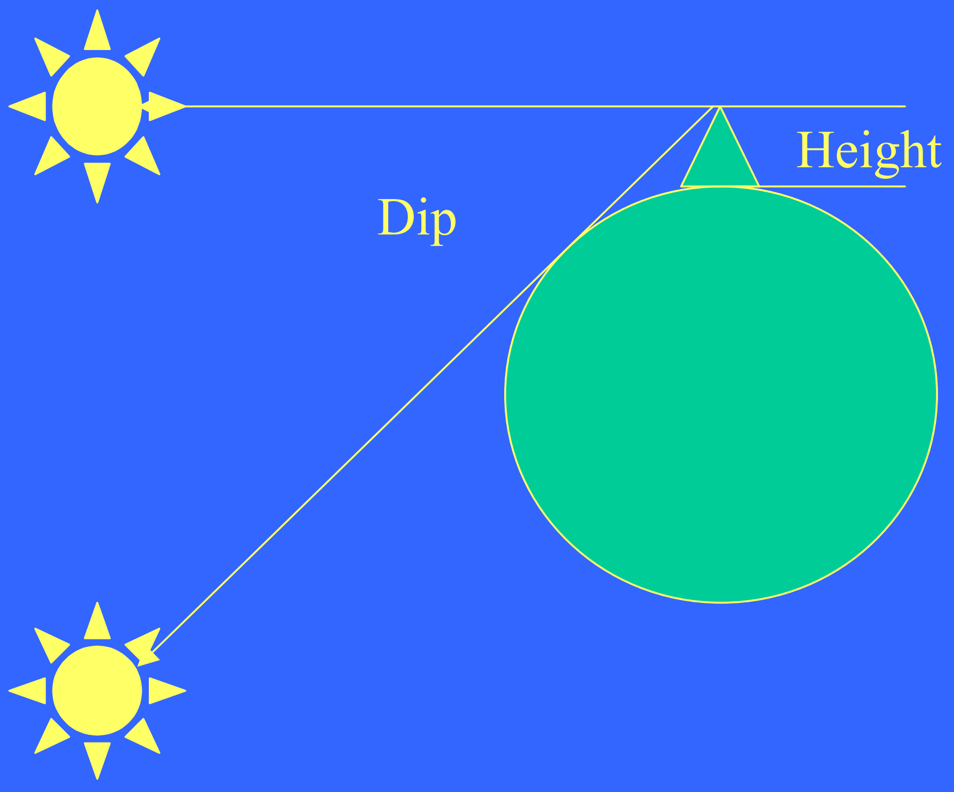
Sunset Path



- Reality Corrections
  - Semidiameter 16’
  - Refraction 34’
- Time Correction
  - 50’ x 4 / Sin Ψ
- Azimuth Correction
  - 50’ x Cos Ψ

Walking Shadow Designs

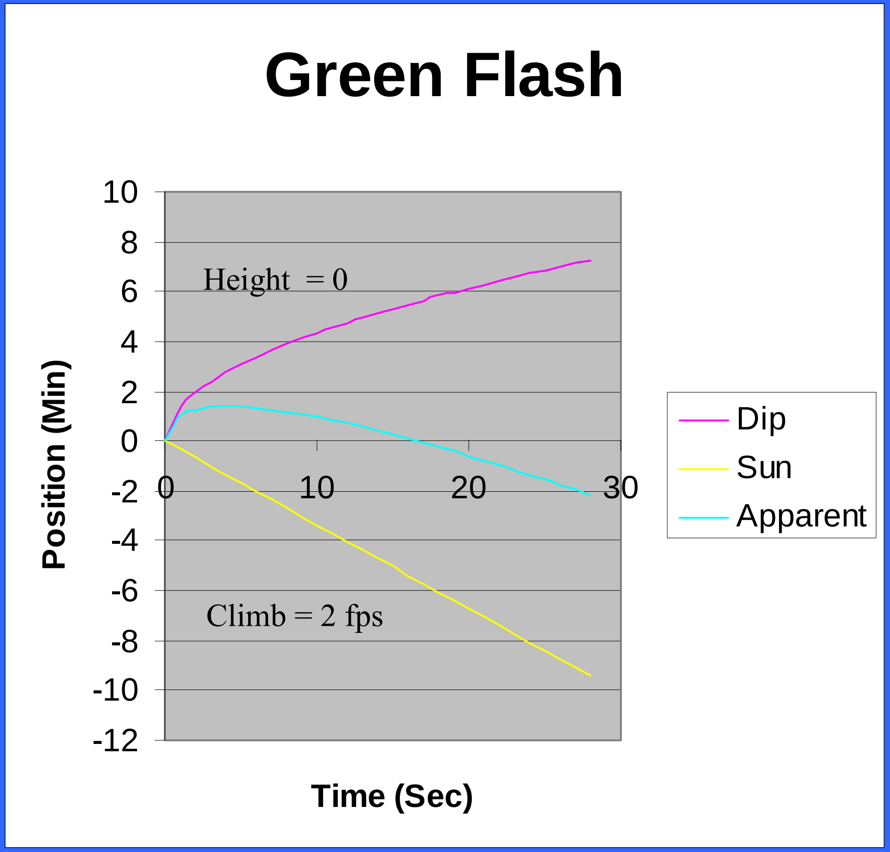
Dip Correction



- Earth's curvature and the height of observer affect the apparent horizon
- Dip = 0.97 x π height (ft)
- Height Dip
  - 10 ft 3 '
  - 100 ft 9.7'
  - 1000 ft 30'

Walking Shadow Designs

Green Flash



- Have you seen the Green Flash at the instant of sunset?
- Can you climb fast enough to increase the dip and see multiple Green Flashes ?
- Depends on initial height, Ψ and climb rate

Walking Shadow Designs

Sunset Rate

- Sunset rate = Solar rate (¼°/minute) x Sin Ψ
- Equator: Ψ= 90°, the sun always sets at 15'/'
- At Latitude 51°, the sun sets at :
  - Equinox, D = 0, Ψ = 39°, rate is 9.44'/'
  - Solstice, D = 23.44°, Ψ = 32.1°, rate is 7.97'/'
- Time flies when you are having fun!
- Tropical sunsets are half as long!

Walking Shadow Designs

Sunset Phenomena

- Functions of Latitude and Declination solvable with high school trig and a pocket calculator
  - Time: $\cos t = -\tan L \times \tan D$
  - Location: $\cos Z = \sin D / \cos L$
  - Path: $\cos \Psi = \sin L / \cos D$
  - Rate: $\sin \Psi \times \frac{1}{4}^\circ$ / minute (15'/')
- Reality checks applicable

Walking Shadow Designs