

Designing a Sundial from Scratch

Roger Bailey
NASS 2003 Banff

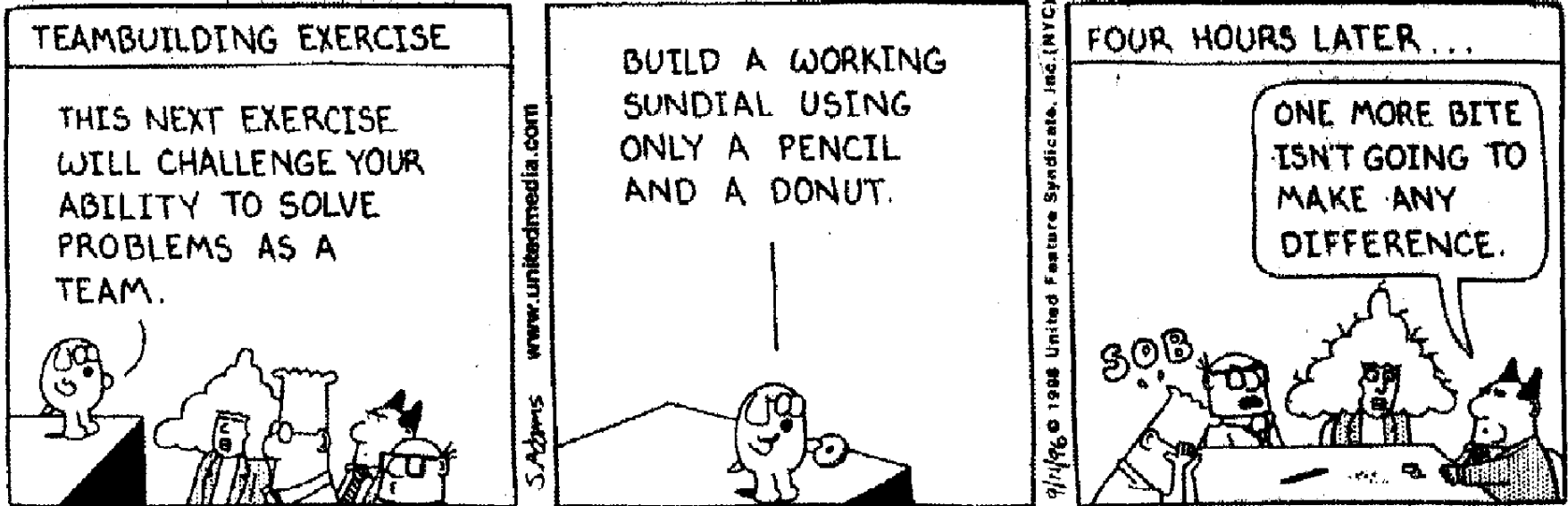
Walking Shadow Designs

Walking Shadow Designs

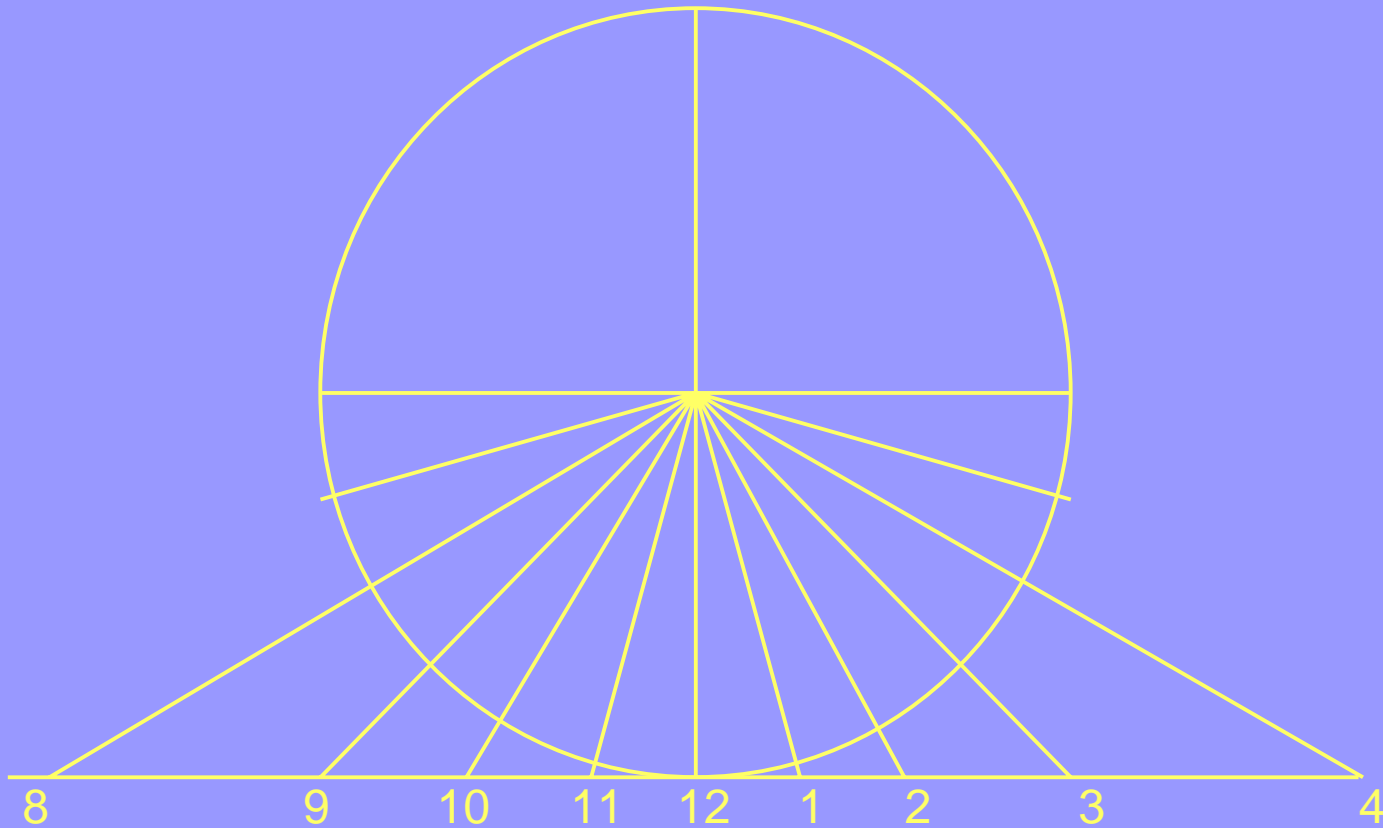
Designing a Sundial from Scratch

- No reference books
- No computers or design software
- No Internet access
- Use Greek geometry: Ruler and Compass
- Use geometric construction
- Extend geometry to derive trigonometry
- Protractor? Trig tables? Calculator?
- Teambuilding exercise?

Dilbert

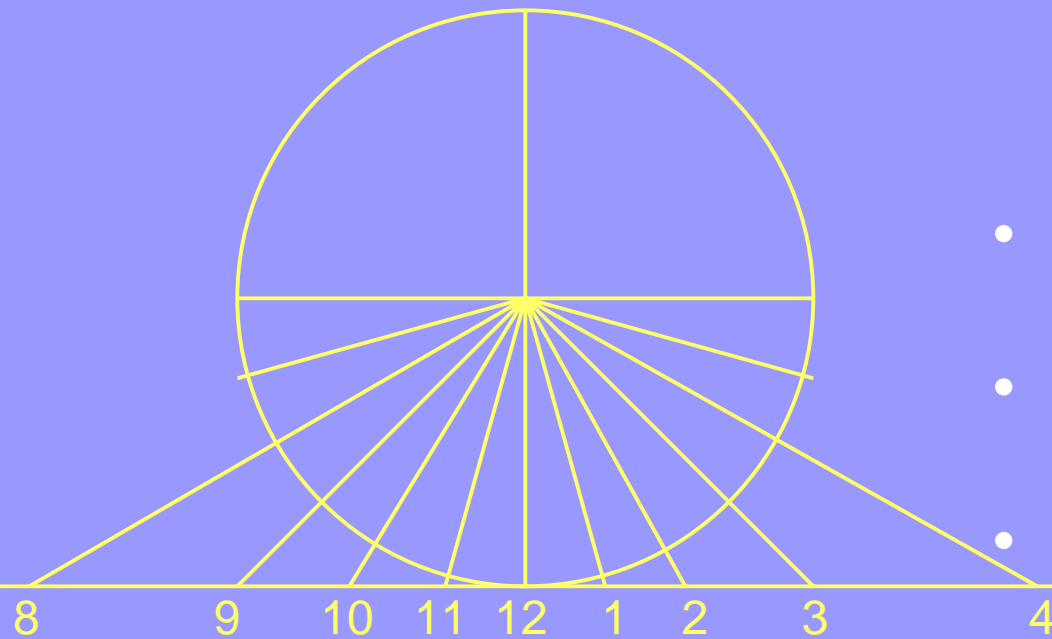


Equinoctial Tangent Line



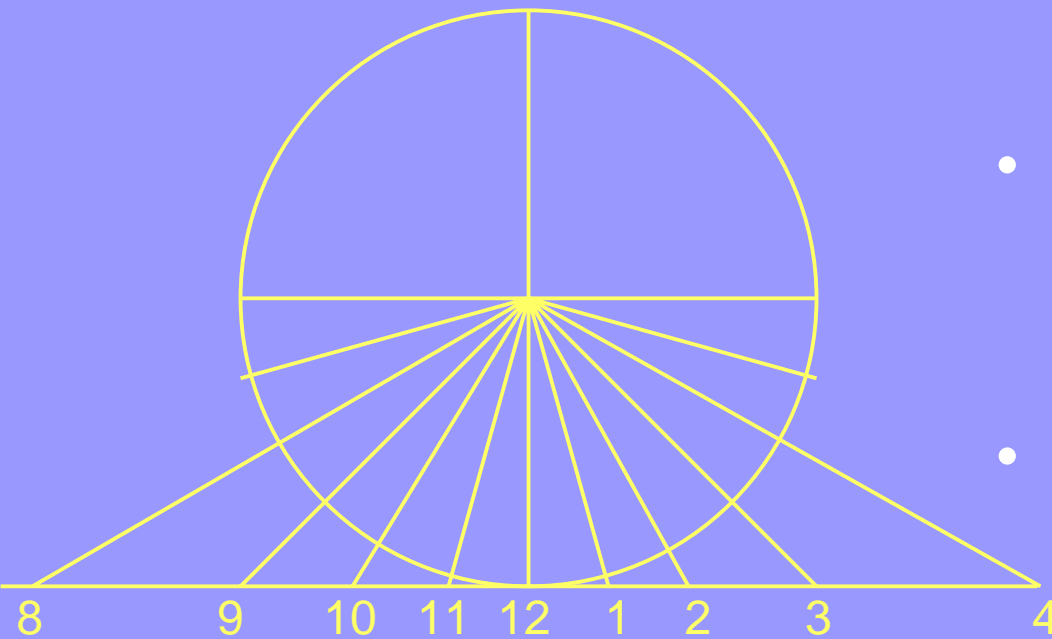
Equinoctial Tangent Line

- Picture an Armillary Sphere or equatorial sundial looking down the polar axis at Equatorial plane
- Equatorial disc as a circle, time angles 15°
- Angles (90, 60, 45, 30, 15) using ruler and compass
- Horizontal Tangent line



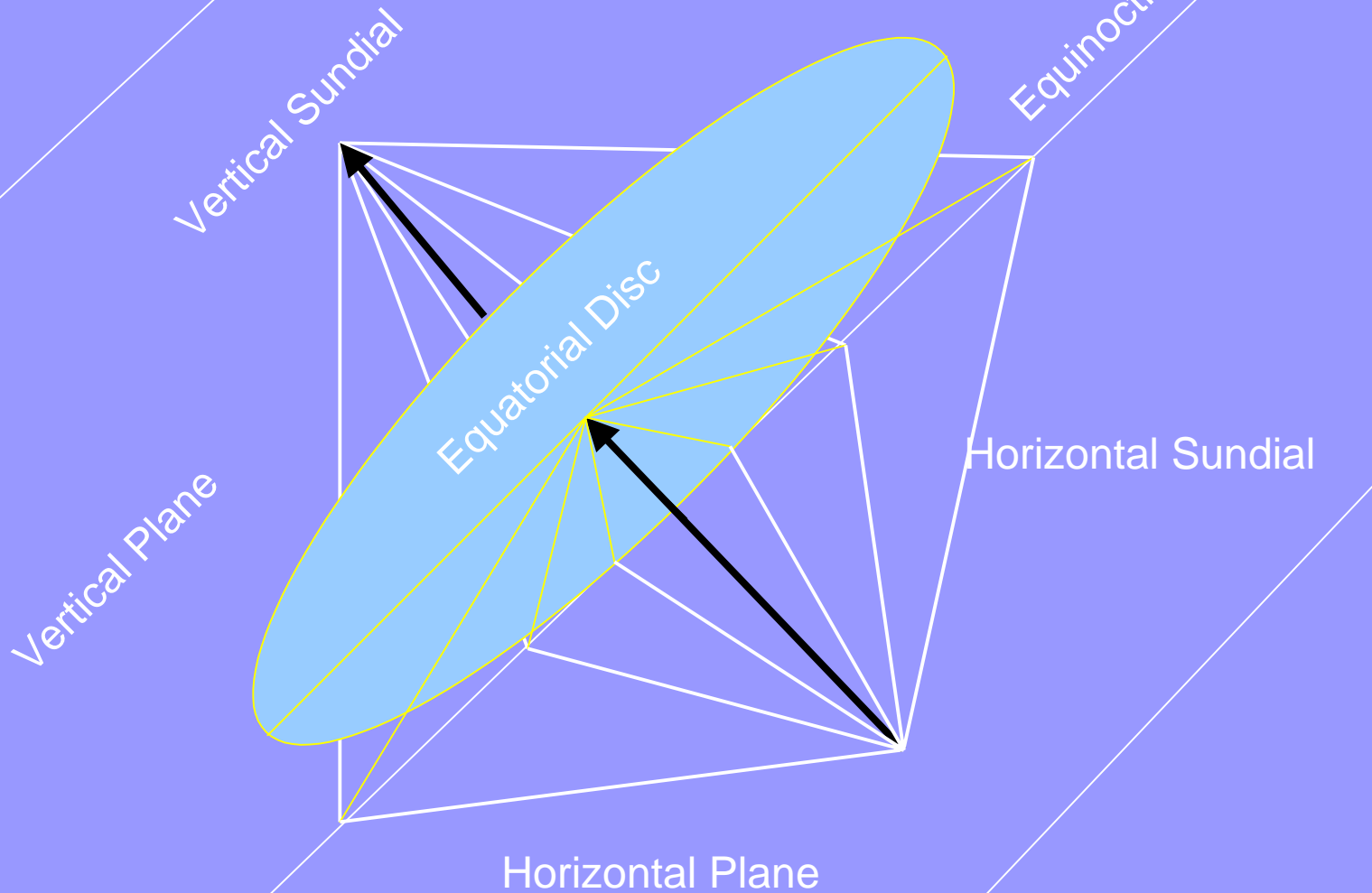
$$d = r \tan t$$

Equinoctial Tangent Line



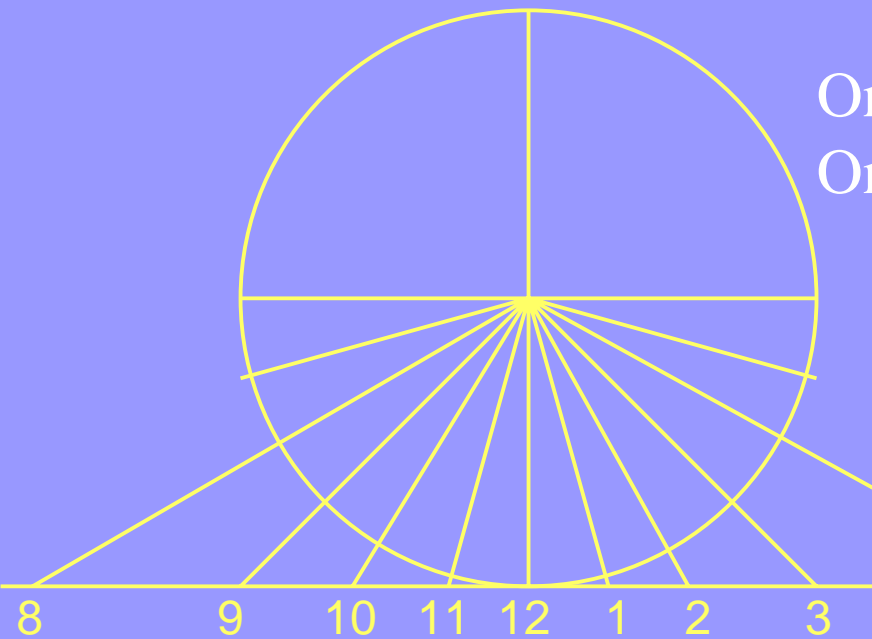
- Equinoctial Line defines the Polar Sundial where the plane is parallel to the gnomon
- Equatorial Line is the intersection line at the base of horizontal and vertical south facing sundials
- The gnomon intersection point and the distances on define the hour angles for horizontal and vertical sundials

Planar Sundials



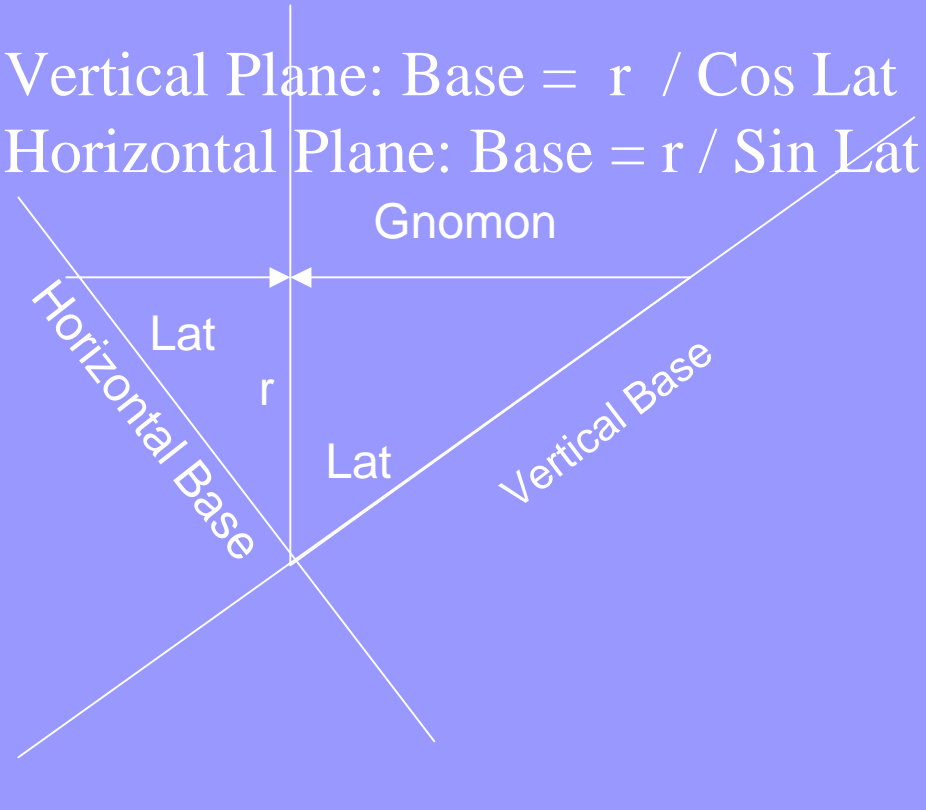
Gnomon Base Lengths

Gnomon Bases on the planes are the hypotenuse of the right angle triangles with one side = r and containing the Latitude.



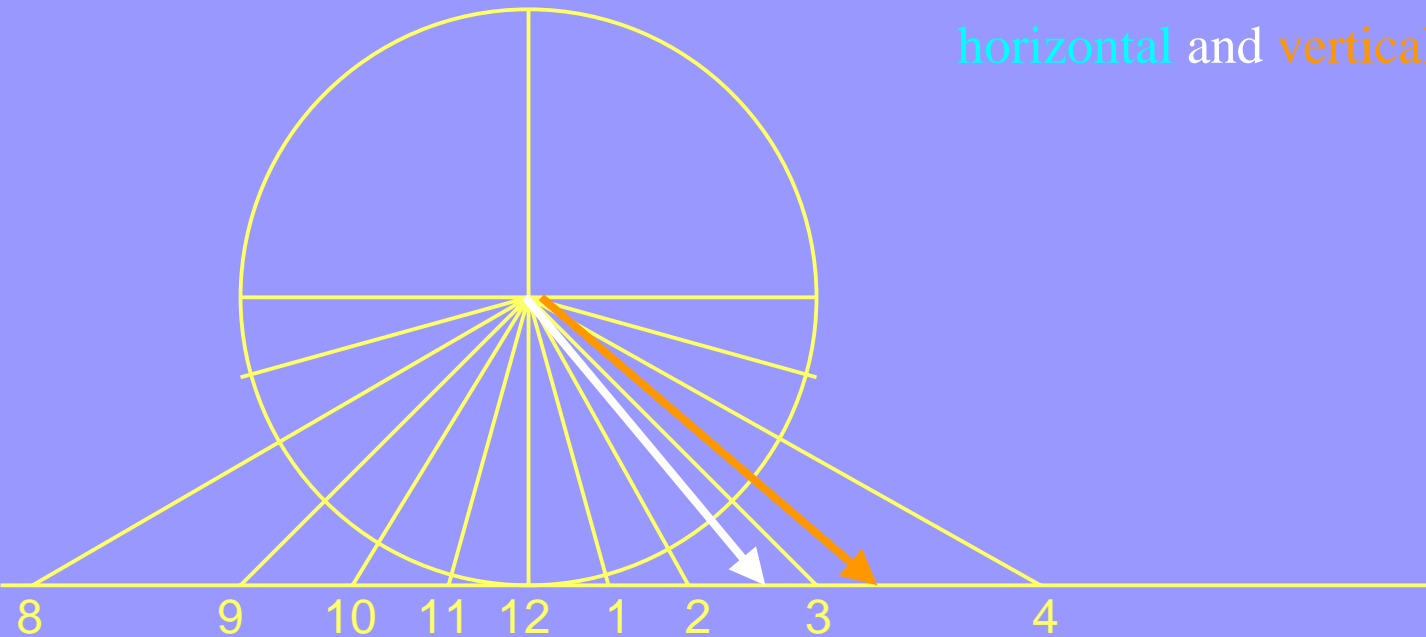
On Vertical Plane: Base = $r / \text{Cos Lat}$

On Horizontal Plane: Base = $r / \text{Sin Lat}$



Gnomon Base Lengths

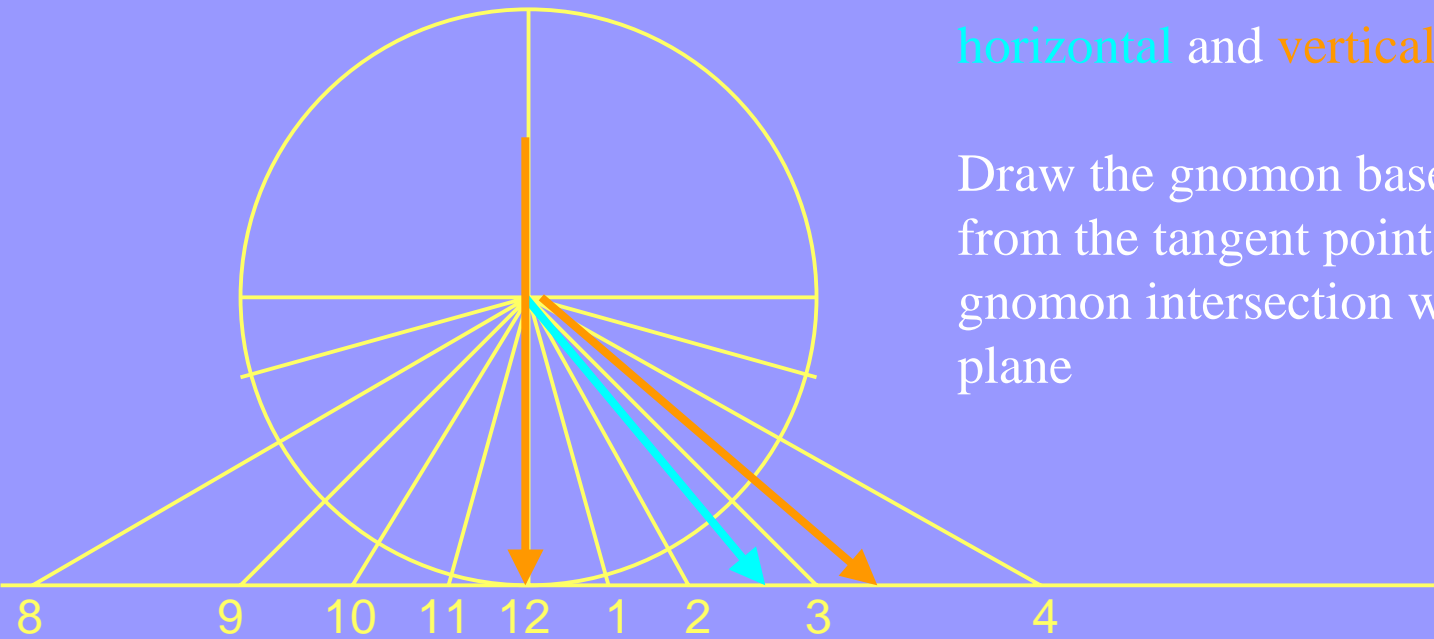
Use the tangent line and r to find the gnomon base lengths for the latitude and co-latitude for **horizontal** and **vertical** dials.



Gnomon Base Lengths

Use the tangent line and r to find the gnomon base lengths for the latitude and co-latitude for **horizontal** and **vertical** dials.

Draw the gnomon base vertically from the tangent point to set the gnomon intersection with the plane

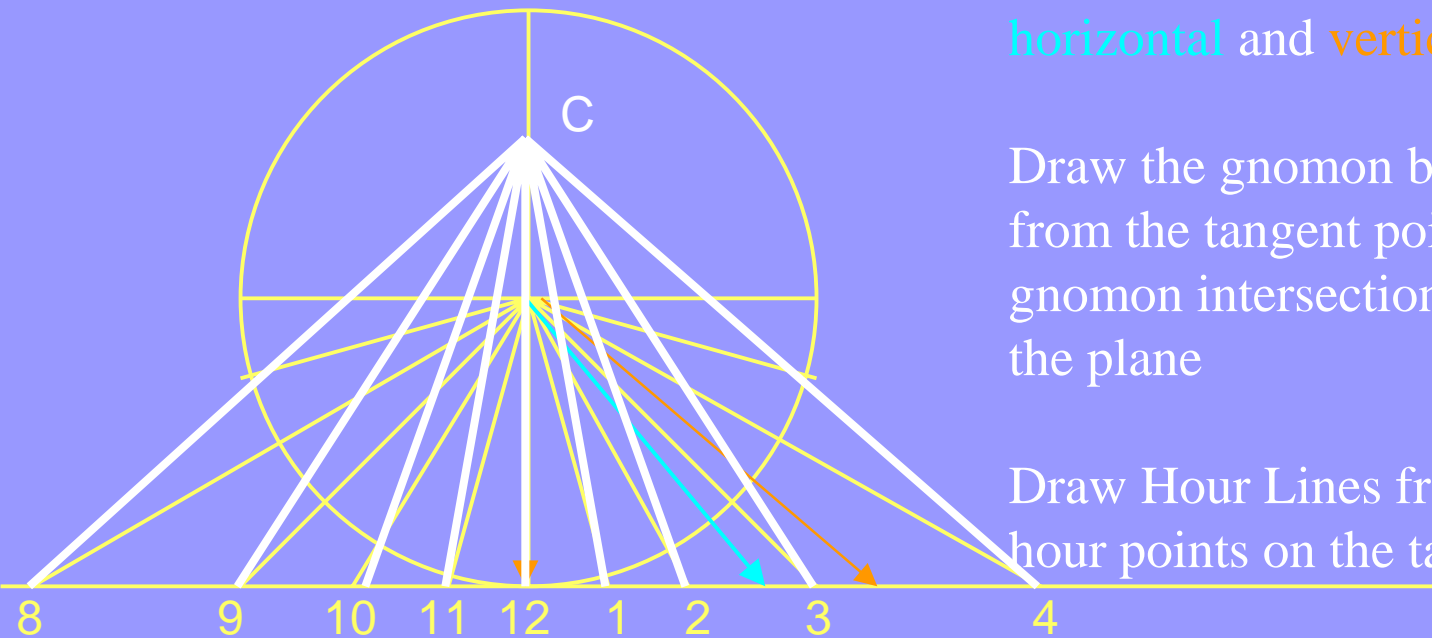


Hour Lines

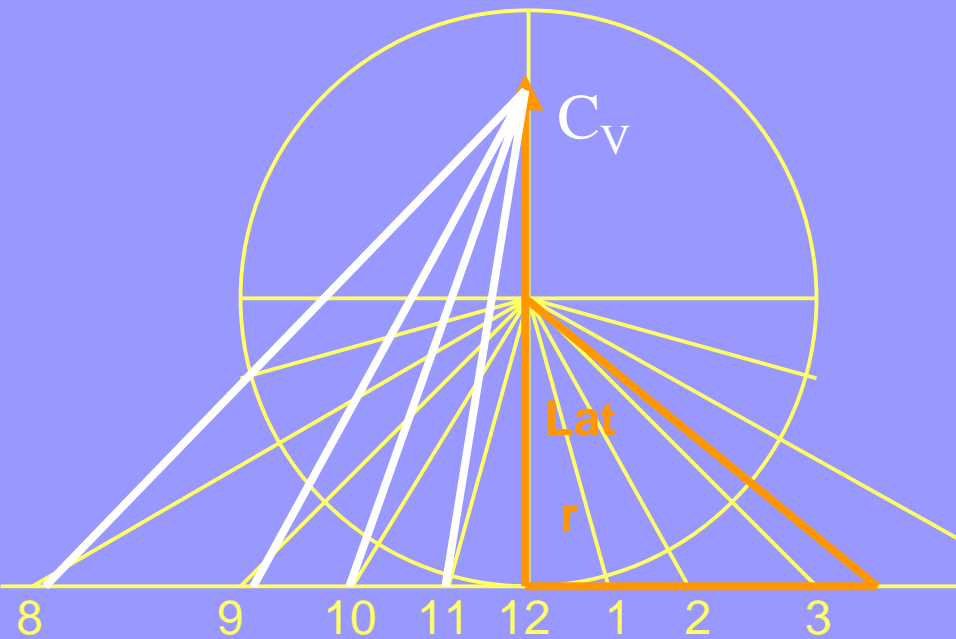
Use the tangent line and r to find the gnomon base lengths for the latitude and co-latitude for **horizontal** and **vertical** dials.

Draw the gnomon base vertically from the tangent point to set the gnomon intersection point C with the plane

Draw Hour Lines from C to the hour points on the tangent line

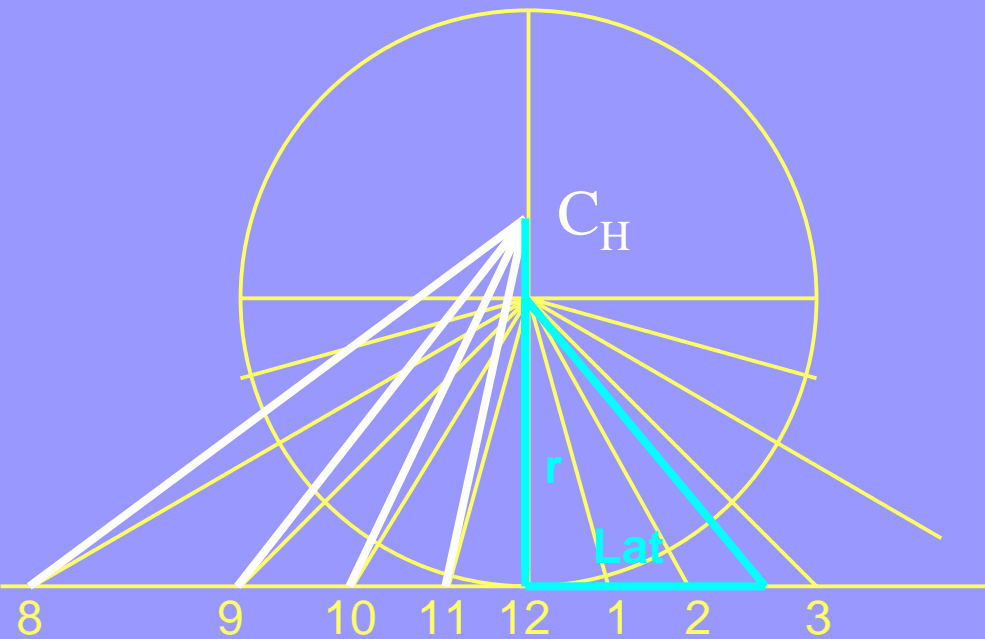


Vertical Sundial



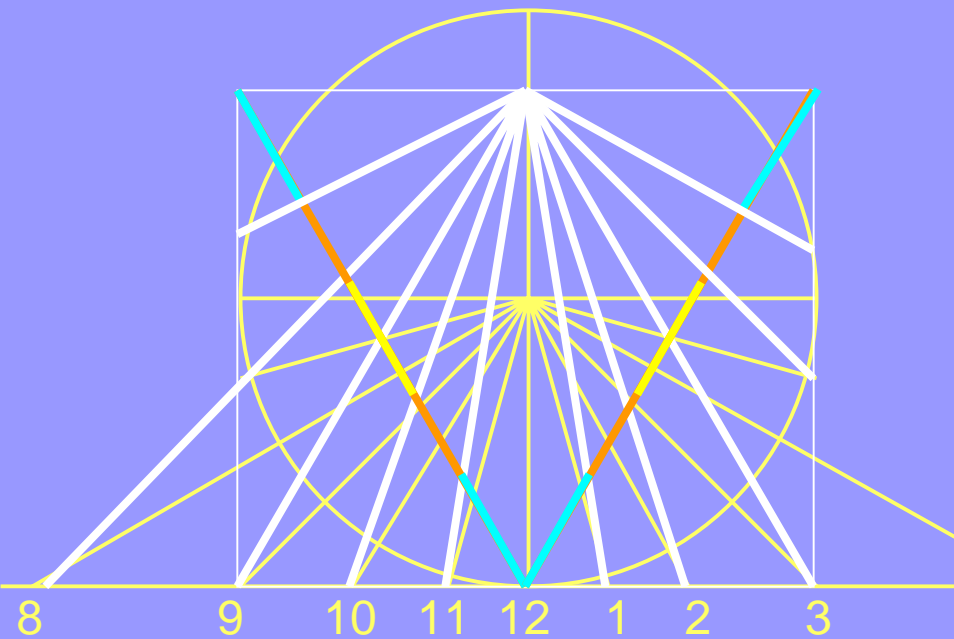
- Draw right angle triangle to determine gnomon base length. Latitude angle at centre of disc, radius is from centre to tangent point
- Hypotenuse (Gnomon Base) is $= r / \text{Cos Lat}$. Draw to set gnomon intersection point C_V with vertical plane
- Draw Hour Lines from base to tangent line distances
- $\text{Tan HA} = r \text{Tan } t / \text{base}$ or
- $\text{Tan HA} = \text{Tan } t \times \text{Cos Lat}$

Horizontal Sundial



- Draw right angle triangle to determine gnomon base length. Latitude angle at tangent line; radius is from disc centre to tangent point
- Hypotenuse (Gnomon Base) is $= r / \sin \text{Lat}$. Draw to set gnomon intersection point C_H with horizontal plane
- Draw Hour Lines from base to tangent line distances
- $\tan \text{HA} = r \tan t / \text{base}$ or
- $\tan \text{HA} = \tan t \times \sin \text{Lat}$

Problem with Infinity



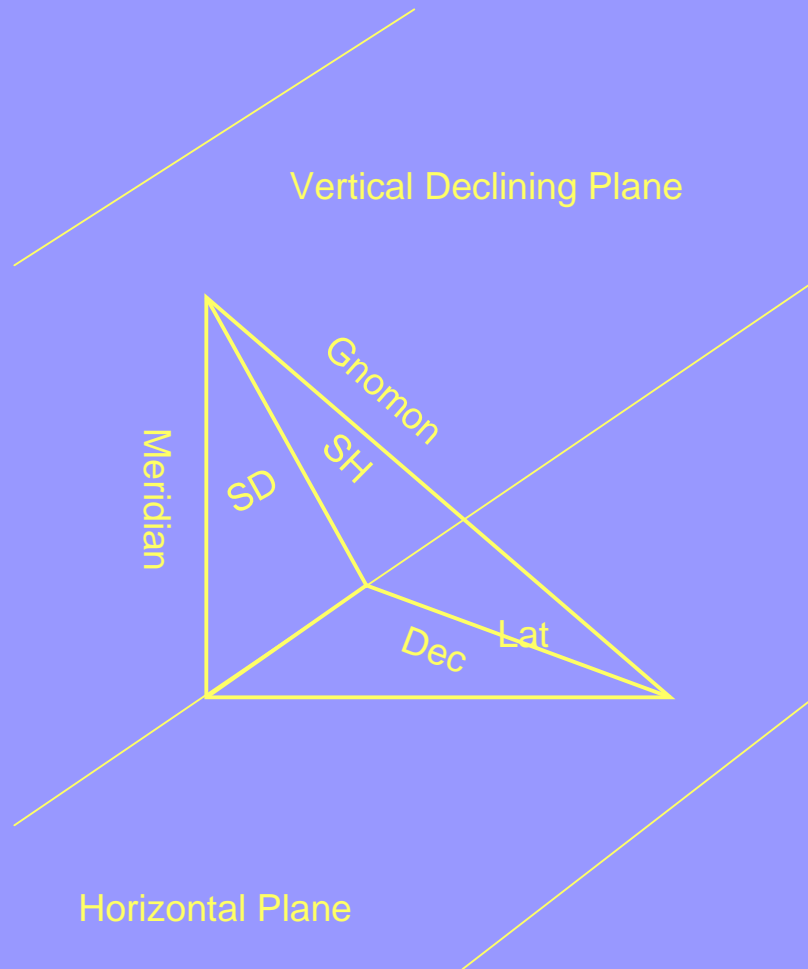
- Six o'clock line is infinite but defined
- 7 am and 5 pm are off scale
- Turn the corner at 9 am and 3 pm (45°)
- Box the circle through gnomon base point
- Scale from diagonal line (Waugh's technique)

Finish the Dial



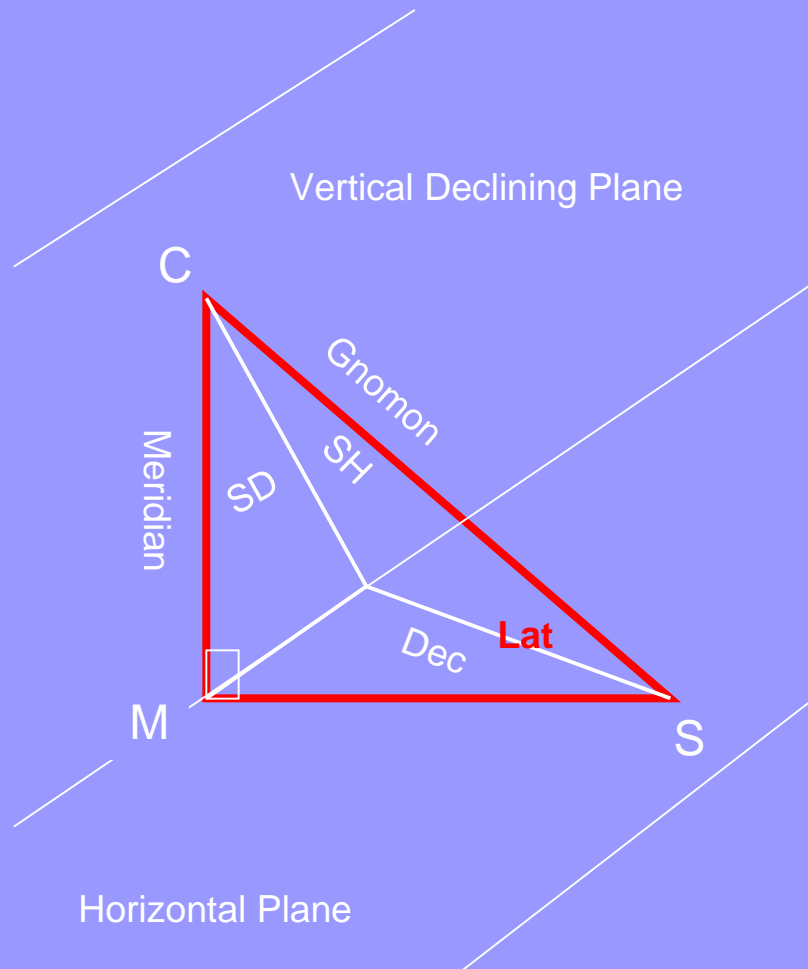
- Delete the construction lines
- Add a border and frame
- Trim the hour lines
- Add the numbers, text and decoration

Vertical Declining Sundial



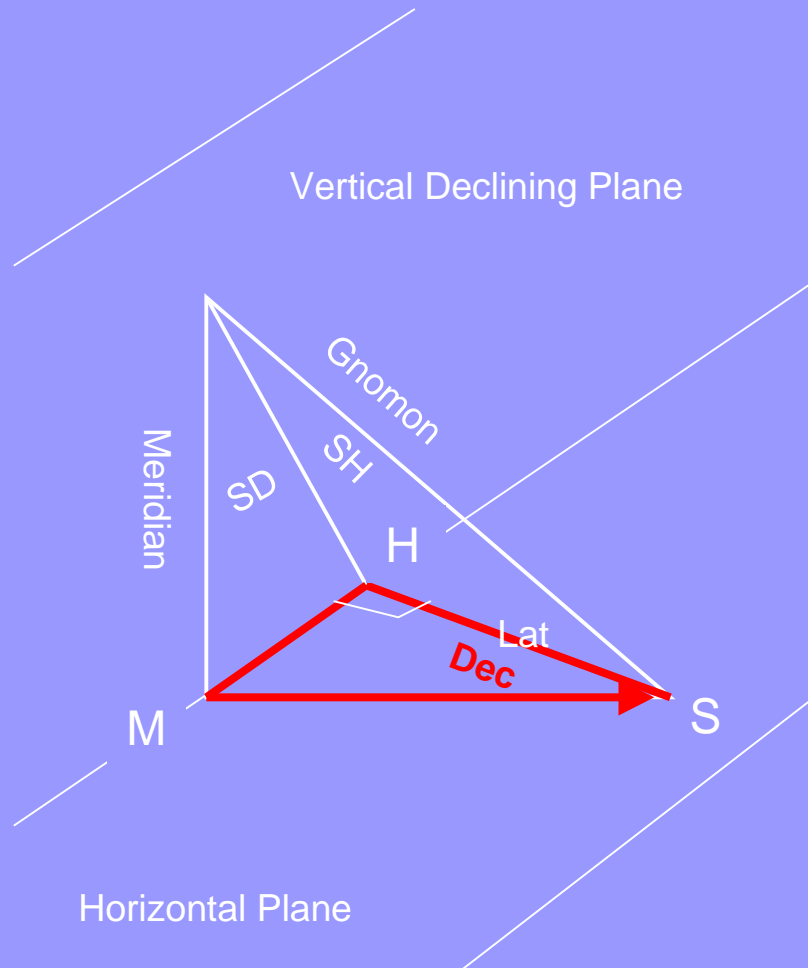
- Treat declining sundial as south facing equivalent
- Solve VD Tetrahedron for Substyle Distance SD and Substyle Height SH
 - Vertical Meridian Plane
 - Horizontal Declination Plane
 - Vertical SD Plane
 - Sloped SH Plane

Meridian Plane: Latitude Triangle



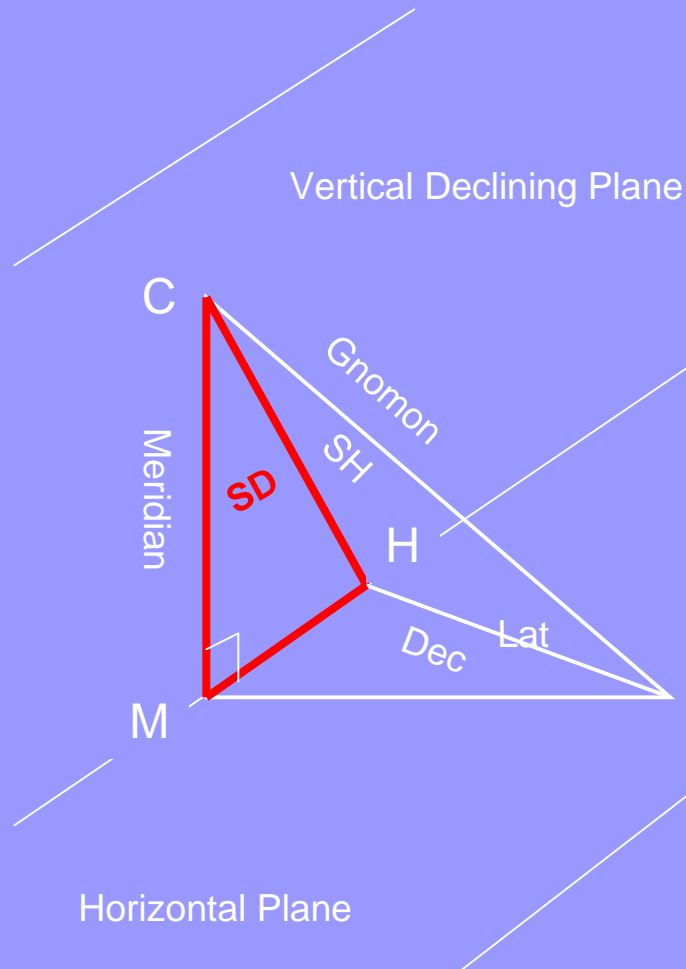
- For all vertical planes
 - Gnomon points south
 - Noon shadow is vertical
- For Meridian Plane
 - Gnomon is true length
 - Gnomon angle with horizontal is true Latitude
- $CM = G \times \text{Sin} (\text{Lat})$
- $MS = G \times \text{Cos} (\text{Lat})$

Horizontal Plane: Declination Triangle



- $MH = MS \times \text{Sin}(\text{Dec})$
 $MH = G \times \text{Cos}(\text{Lat}) \times \text{Sin}(\text{Lat})$
- $HS = MS \times \text{Cos}(\text{Dec})$
 $HS = G \times \text{Cos}(\text{Lat}) \times \text{Cos}(\text{Dec})$

Vertical Declining Plane

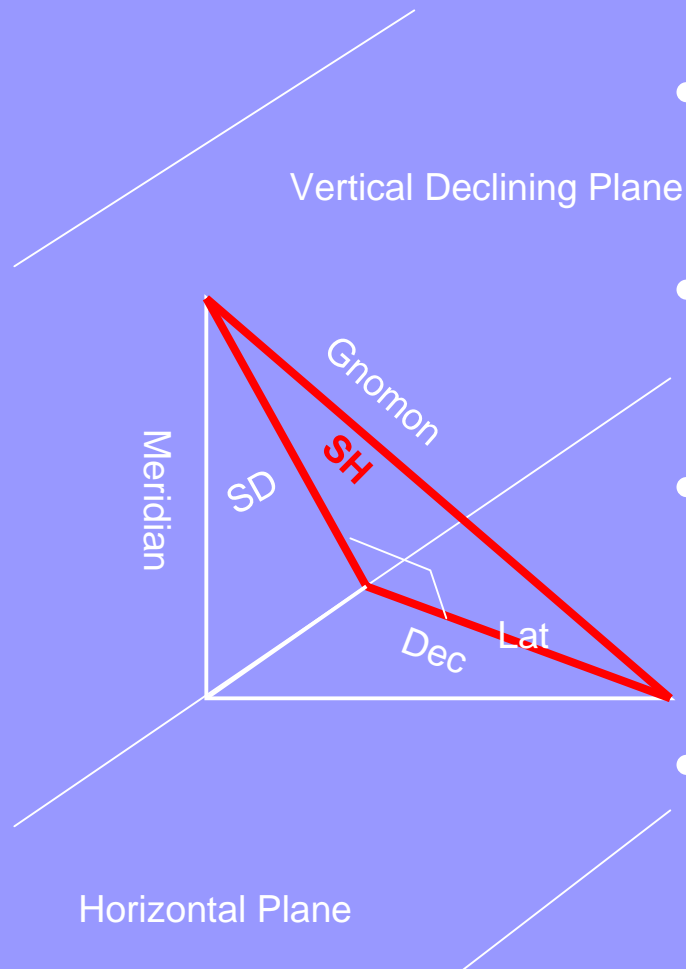


- $\text{Tan (SD)} = \text{MH} / \text{CM}$

- $\text{Tan(SD)} = \text{G} \times \text{Cos(Lat)} \times \text{Sin(Dec)} / (\text{G} \times \text{Sin(Lat)})$

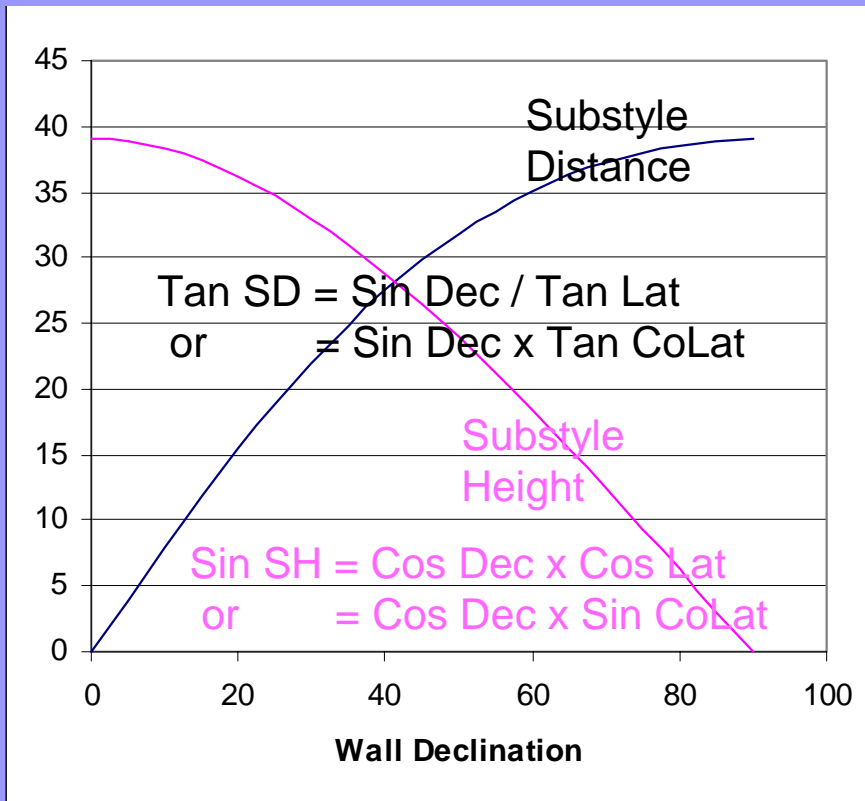
- $\text{Tan(SD)} = \text{Sin(Dec)} / \text{Tan(Lat)}$

Inclined Declining Plane



- For Substyle Height SH
- $\text{Sin}(SH) = HS / G$
- $\text{Sin}(SH) = G \times \text{Cos}(\text{Lat}) \times \text{Cos}(\text{Dec}) / G$
- $\text{Sin}(SH) = \text{Cos}(\text{Lat}) \times \text{Cos}(\text{Dec})$

Substyle Distance & Height

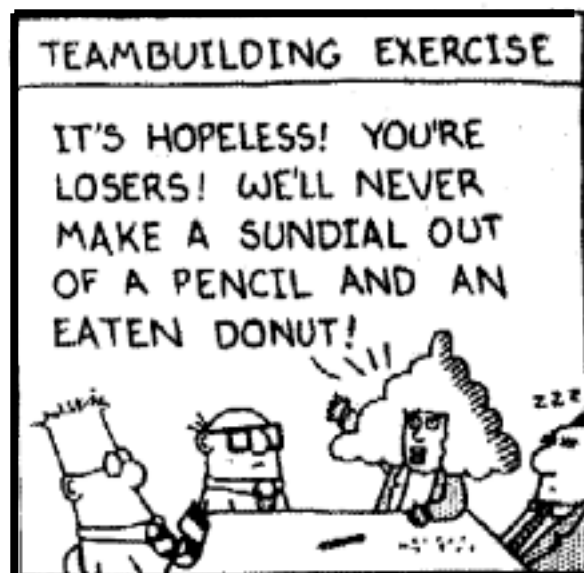


- Substyle Distance increases with wall declination from zero to Co-Latitude
- Substyle Height decreases from Co-Latitude to zero with wall declination

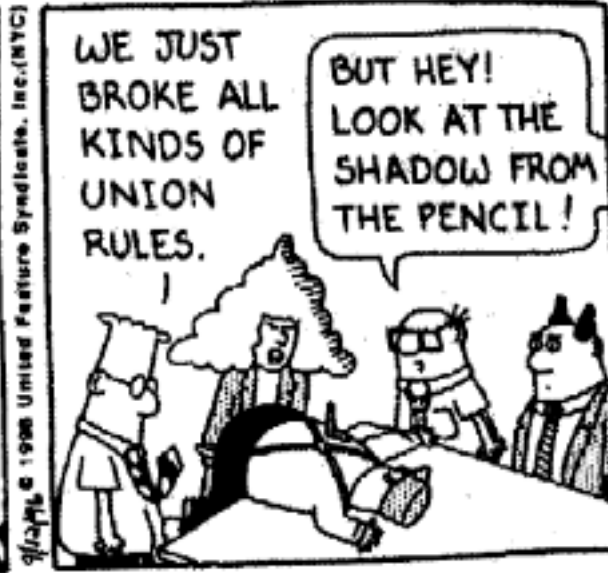
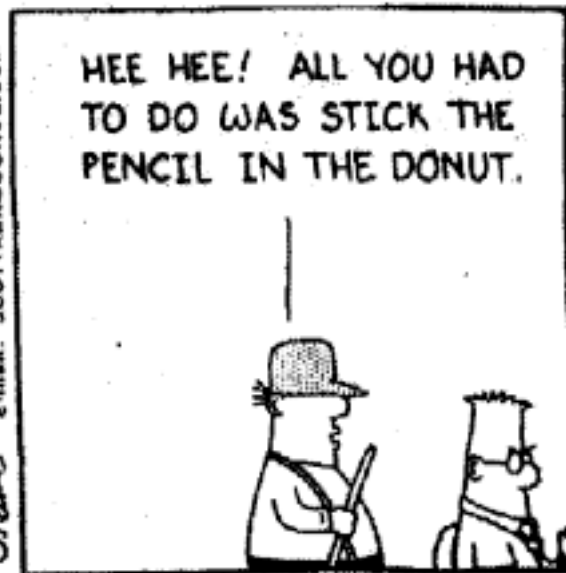
Sundials from Scratch

- Draw circle and Tangent Line
- Draw time segments to Tangent Line
- Find gnomon base point using the tangent line
- Draw Hour Lines from base point to tangent points
- Scale from diagonal for hours near 6 o'clock
- For vertical declining dials, first solve VD tetragon for Substyle Distance and Height

Dilbert



3/2/98 E-mail: SCOTTADAMB@AOL.COM



© 1998 United Feature Syndicate, Inc. (NYC)