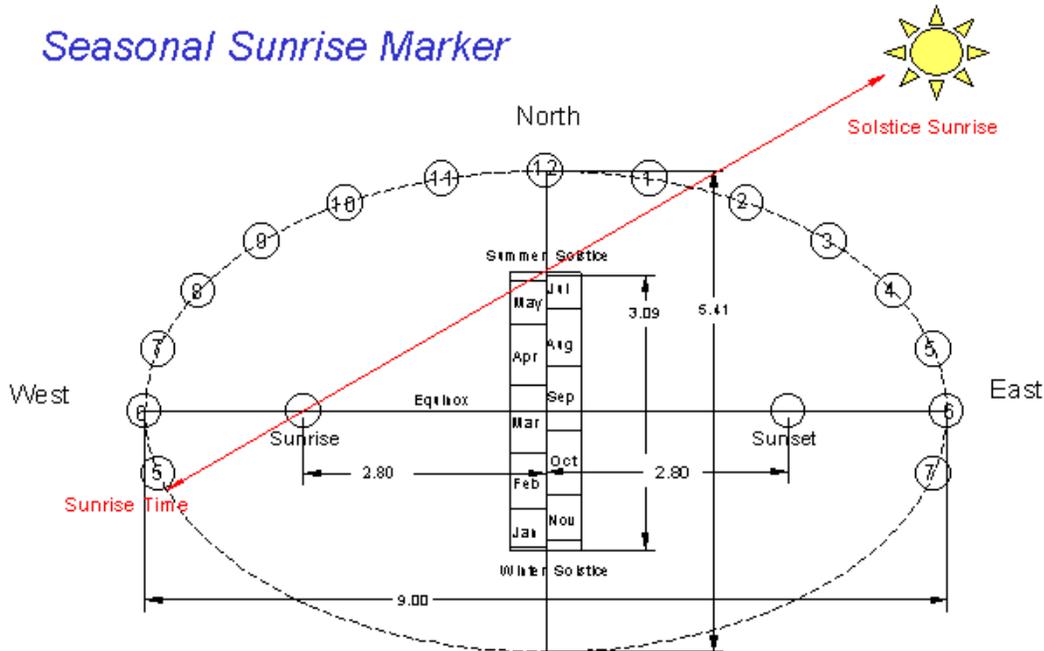# Seasonal Markers for Analemmatic Sundials
Roger Bailey, Walking Shadow Designs, Canmore Alberta, Canada

Good questions can lead to interesting discoveries. This was the case when Mike Deamicis-Roberts asked on the sundial mailing list this question:*"Can a sundial be used to follow the sun through the seasons?"* This led to the discovery of seasonal sunrise and set markers for analemmatic sundials. This article outlines the discovery process: the concept of seasonal markers, mathematics, applications, alternatives, errors and corrections.


This sketch outlines the concept of seasonal markers. These are points on the east west axis of analemmatic sundials that can be used with the dial's zodiac or date line on the north south axis to determine where and when the sun rises and sets. The drawing illustrates the summer solstice sunrise but the technique works for any date. Stand on the sunrise mark and sight over the date on the Zodiac to see where on the horizon the sun rises. Stand on the date and sight over the sunrise marker to see the time of sunrise.

Let's follow Mike's questions that led to the development of this concept, starting with the basic question: *"Can a sundial be used to follow the sun through the seasons?"*
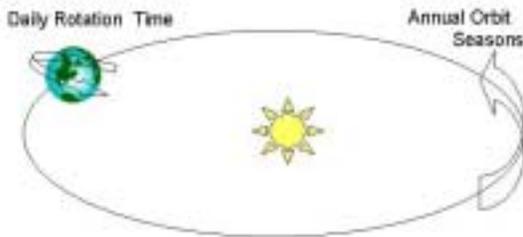

We are all familiar with the cycles of the sun due to daily rotation of the earth and the annual orbit of the earth around the sun. The main function of a sundial is to mark this daily rotation, 15 degrees per hour and display it as time. Time is recorded in many ways with sundials but the essence remains. The daily rotation is divided into hours.

Altitude and Declination: The annual cycle of the earth around the sun gives us another unit of time, the year divided into seasons. The tilt of the earth's axis of 23.5 degrees with respect to the orbital plane results in changes to the relative position of the sun, a function that we call solar declination. This change with the seasons of the year is generally noticed as changes to the altitude of the sun and the length of the day.

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Altitude of the Sun: The altitude of the sun at noon is equal to the Co-Latitude plus the solar declination.
Alt = 90 - Lat + Dec.
For a typical US location at latitude 40 degrees, the altitude of the noon sun will vary from 73.5 degrees at the summer solstice to 26.5 degrees at the winter solstice

Sundial Declination Lines

Equinox

Solstice
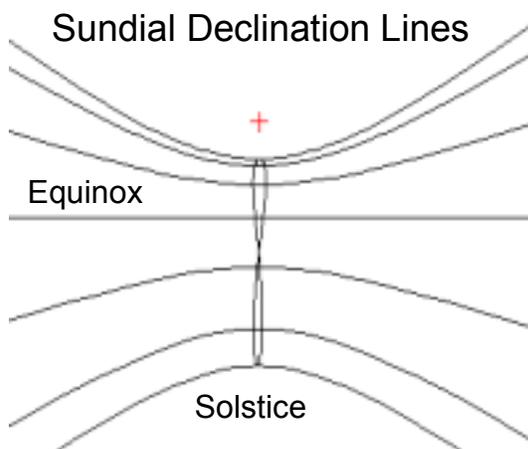
Sundials can track the seasons in several ways. The length of the shadow varies with the altitude of the sun that changes with solar declination. Declination lines that track the length of the tip of the gnomon shadow can be added to dial faces to show the date. Although declination lines can be drawn to note any day of the year, they are most often drawn to show the extreme positions, the equinoxes and solstices.

The location and time of the rising and setting sun is also a good way to track the seasonal solar cycle. Marks can be added to the dial to indicate the time and direction of sunrise and set. Distant marker points can be used to indicate where on the horizon the sun rises and sets. These techniques are often used to indicate specific dates like the solstices or equinox. Many ancient cultures used horizon markers to track the seasons. Stonehenge and similar megaliths in Europe are the classic examples of this technique, as are some of the "Medicine Wheels" on the western plains of North America.

Sunrise and Set: For any date and location are simply determined from the Latitude (Lat) and Solar Declination (Dec).

The time of sunrise and set, as the time angle (t) from solar noon is:
Cos t = Tan Lat x Tan Dec.

The azimuth (Az) of the sun at an altitude of zero is:
Cos Az = Sin Dec / Cos Lat.

Mike Deamicis-Roberts was interested in the suggestion to build an analemmatic dial with horizon marker points aligned to specific sunrise points but he wondered if the markers could be incorporated into the dial itself. His second good question was: "*Is*

there a point in an analemmatic sundial that can be used with the Zodiac date line to show where the sun rises?" The geometry of the analemmatic dial date line and the azimuth of sunrise is shown in the following sketch.
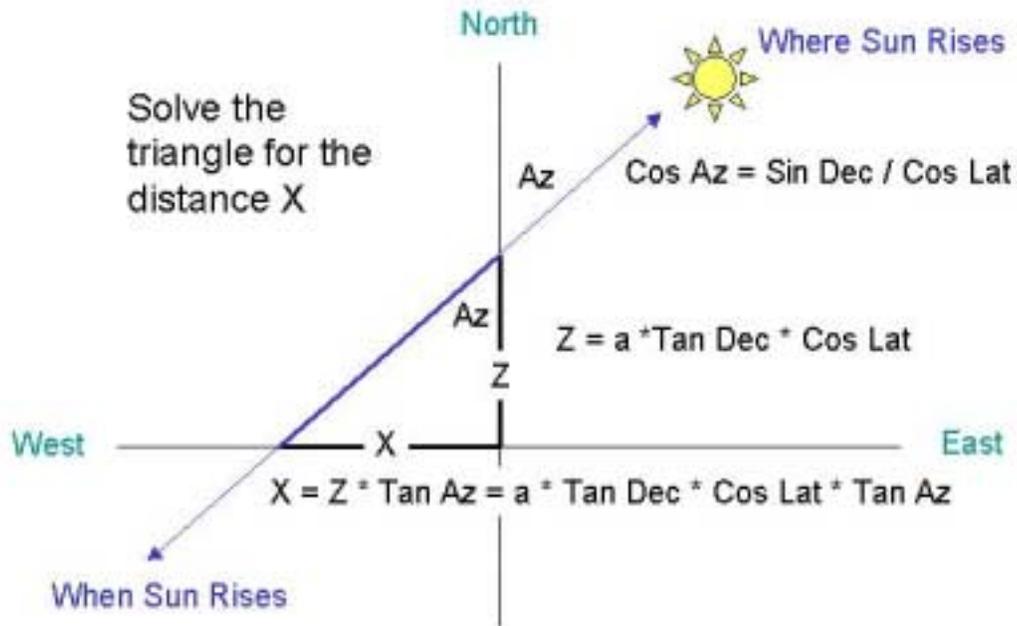
The seasonal marker point is at distance X along the east west axis. This is determined from the Azimuth angle (AZ) and the distance Z on the north south Zodiac date line. For analemmatic dials this date point is Z = a x Tan Dec x Cos Lat where a is the size parameter, the length of the semi major axis. The tangent of the azimuth angle Az is X/Z, so the seasonal marker distance X = a x Tan Dec x Cos Lat x Tan Az. Solve this equation for various values of solar declination to see if the marker distance X is the same through the year. The results are shown below for Mike's location, Latitude 36.9°North. The lines for different dates converge almost to a point. The seasonal marker concept is valid.
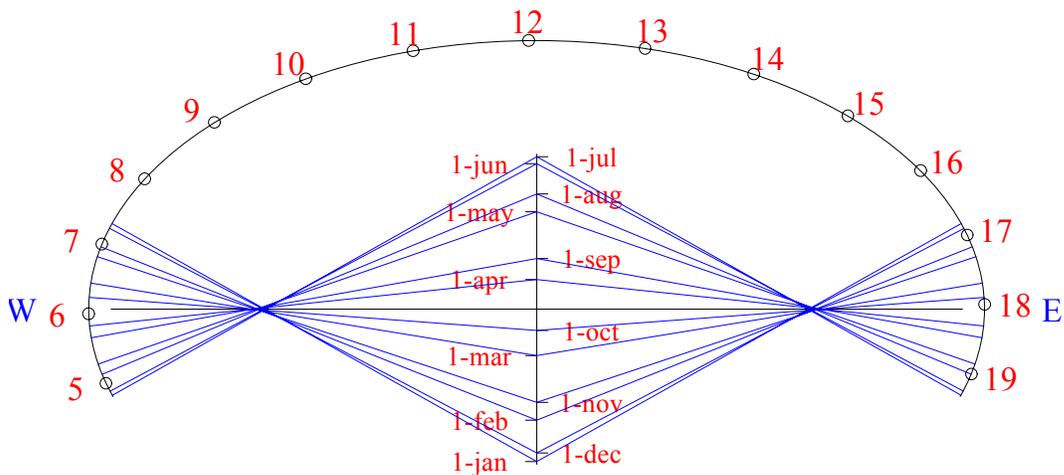
Mike Deamicis-Roberts built the first dial to incorporate Seasonal Markers. In this picture of his dial, Mrs. Deamicis-Roberts is pointing over the seasonal markers to the time of sunrise and sunset. The sweep of her arms indicates the length of the day at that date and location. This shows the utility of the Seasonal Markers for public dials. The concepts of time, seasons, length of the day, sunrise and sunset time and location are all easily demonstrated.


The concept is not limited to large dials with a person as the gnomon. It is also applicable to smaller analemmatic dials as show by this engraved stone dial by John Carmichael of Tucson Az. http://www.sundialsculptures.com

Although the Seasonal Marker concept is valid and useful, it is an approximation. The seasonal marker lines do not converge to aa single point. There is some variation in the seasonal marker distance. This variation is due to the linear approximation of the spherical trigonometry.

 Fer De Vries pointed out that the exact solutions are circles that pass through the date point of the Zodiac, the hour point on the analemmatic ellipse, and the focal points of the ellipse. These day circles or Lambert Circles are shown in green in the following sketch. The straight seasonal marker lines are shown in blue. Both concepts are valid. One is exact, the other an approximation. Both show the time of sunrise and sunset. The seasonal marker lines are better to the where the sun rises and sets.
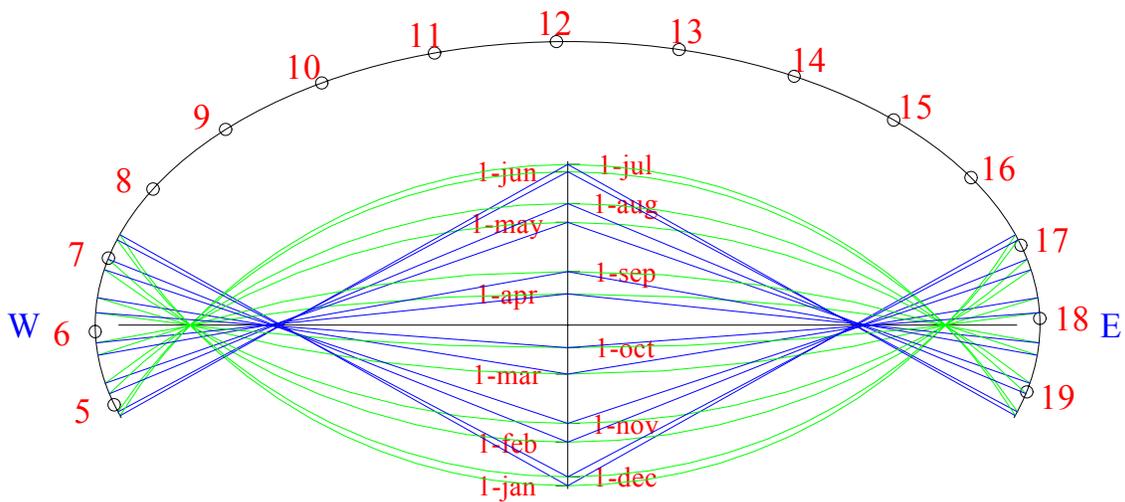

Zooming in on the focal and marker points shows the difference between the Seasonal Markers and Lambert Circles. The circle at the Seasonal Marker shows the range of marker distances through the seasons.

Variation with Latitude
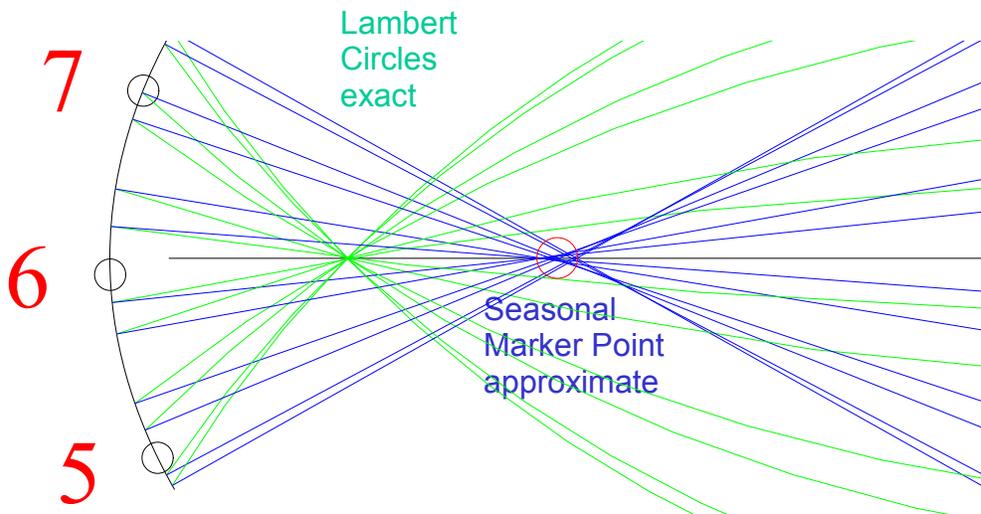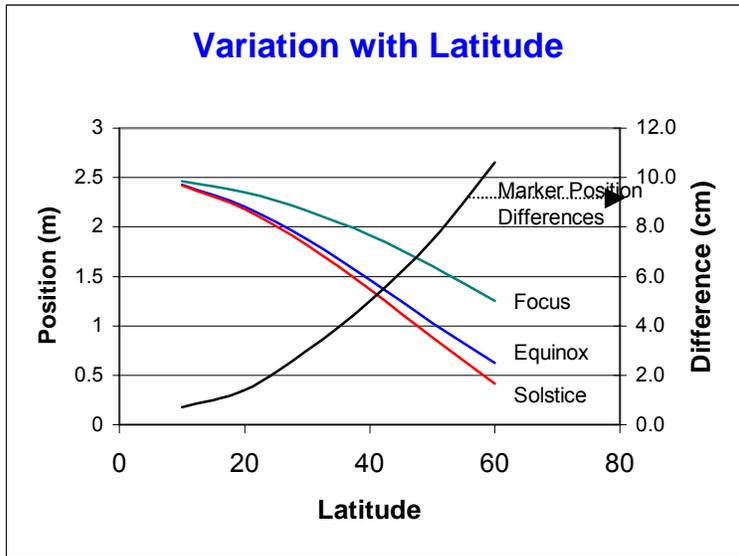
The variation in seasonal marker distance increases with latitude. The dials pictured on the previous pages were designed for low latitudes, 36.9º and 32º. For a typical large analemmatic dial, 5 meters in diameter, there is little difference, 3 to 4 cm, between the marker distance for extreme points, the equinox and solstice. At higher latitudes the error becomes noticeable, increasing to 7.5 cm.

The variation of marker distance with the date is shown below. The periodic nature is obvious going through two cycles per year as the solar declination, (blue line, left axis) goes through one cycle per year. This suggests a way to correct the marker distance using what I have termed an epicycle. As shown below, a circle with the date marked around the circumference can be used to determine the marker distance each day.

Variation with Date

Correction Epicycle
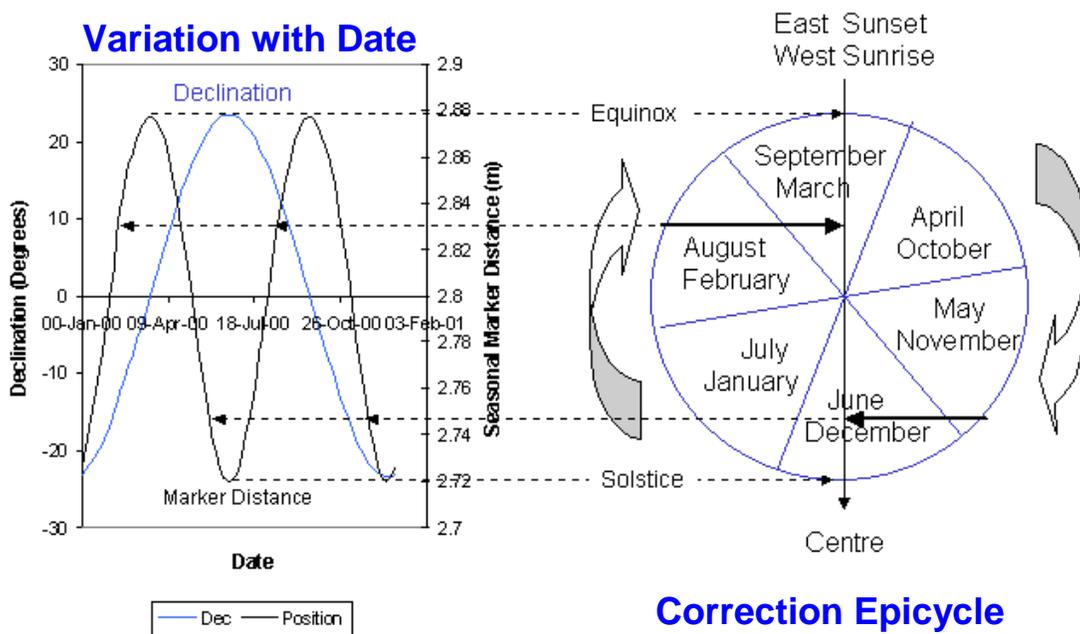
Going around the epicycle, twice per year, generates the sine curve. Drop a perpendicular from the date on the circumference to the axis to find the exact marker position.

To determine the position and diameter of the correction epiclyce, calculate the marker position for the extreme conditions, the equinox and the solstice from the equations outlined previously. Since the Declination is zero at the equinox, and the sine and tangent of zero are zero, use a small value like 0.01 degrees to avoid the problems with zero.

X = a x Tan Dec x Cos Lat x Tan Az where Cos Az = Sin Dec / Cos Lat

The diameter of the correction epicycle is the difference between the marker distances at the solstice and the equinox. The epicycle is placed at the average distance between these two marker distances.

If precise determination of the time and direction of sunrise and sunset are required, the correction epicycle can be included in the design. For most practical designs, the correction is not required and adds an unnecessary complication. A Seasonal Marker disc the size of the correction epicycle would generally be an appropriate marker.

The discussion thus far has related to theoretical sunrise and sunset when the calculated altitude of the centre of the sun is zero degrees. This is not the observed sunrise phenomenon, the first crack of sunlight on the horizon Even with a true horizon, corrections must be made for semi diameter, refraction and dip.

Reality Checks:
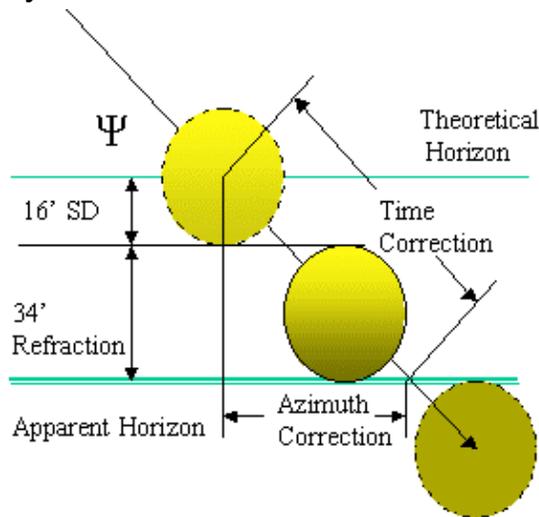
The correction for the semi diameter of the sun is typically 16'. For typical atmospheric densities, the refraction correction is about 34'. For sea horizons, the dip correction to compensate for the curvature of the earth is 1.76' x √height of eye in meters.

The time correction and azimuth correction depend on the angle of the sun path with the horizon. This angle ψ varies with latitude and declination.

Cos ψ = Sin Lat / Cos Dec

For most locations the horizon will not be like a level sea horizon. Hills, mountains, buildings etc will obstruct the true horizon. In the case of Mike Deamicis-Roberts, the Diablo Mountains to the east of his location resulted in a local horizon of about 4 to 5 degrees. Mike's third good question was *"Can you offset the seasonal marker to correct for the real horizon?"* The answer is yes but the azimuth shift depends on altitude of real horizon and path of the rising sun. The offset is always to the north but distance depends on sunrise geometry. It is not constant for constant horizon altitude and azimuth shift but depends on the geometry for different seasons.

Offset Geometry
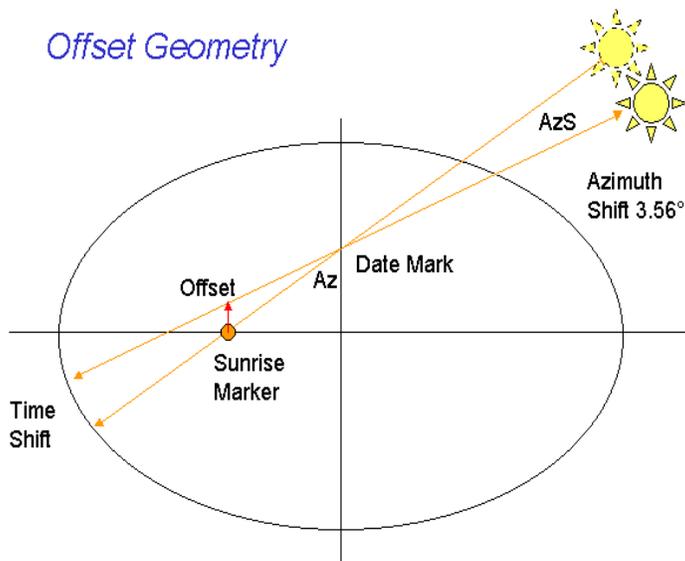
This sketch shows the geometry during the summer. The Azimuth Shift for the horizon at 4.5° is: ft $AzS = Alt / Tan\ \psi$. This gives a typical summer azimuth shift of 3.5°. The offset calculated for the geometry of this 11 meter dial gives an offset of the sunrise marker 22 cm north. Similar calculations for the equinox and winter give offsets of 17 and 23 cm north. An average value of 20 cm, or 8" was chosen for the design.

This picture shows that the offset design was successful. The light of dawn, shining between the legs of the gnomon standing on the Zodiac date mark, frame the offset Seasonal Marker and show the correct time of apparent sunrise.

Conclusions:

• Seasonal Markers are an excellent addition to analemmatic sundials.
• Seasonal Markers are used with the Zodiac Date Mark to show when and where the sun rises and sets throughout the year.
• Markers are linear approximations
• Errors increase with latitude
• Exact solutions are available with the correction epicycle, but are not generally necessary.
• Offsets to correct for the horizon are possible, but not generally necessary.

Associated Software: The following software is available as freeware to assist in the design of analemmatic dials with seasonal markers.

Analemma.xls, an Excel spreadsheet by Roger Bailey and Helmut Sonderegger.
Hor_analem3SM.bas, a Delta Cad BASIC macro by Fer De Vries for analemmatic dials with Lambert Circles, modified by Roger Bailey to draw seasonal marker lines.